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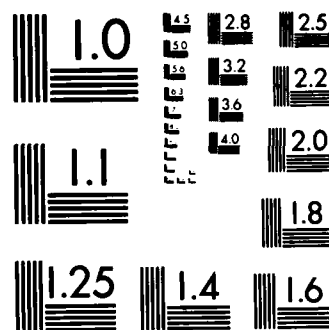
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# Dynasty II, A Nonlinear Implicit Code for Relativistic Electron Beam Tracking Studies

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July 27, 1983

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# DYNASTY II, A NONLINEAR IMPLICIT CODE FOR RELATIVISTIC ELECTRON BEAM TRACKING STUDIES

## I. Introduction

Computational models that are linearized about an axisymmetric equilibrium have proven invaluable in the study of instabilities of propagating beams. For example, linearized (monopole/dipole) particle simulation codes<sup>1</sup> as well as linearized codes with simplified models of beam dynamics<sup>2</sup> have been used for hose instability analysis. However there are other areas of beam propagation phenomenology in which large departure from axisymmetry are essential and inevitable, and in which such linearized models can be quite misleading. For example, in the presence of transverse external magnetic or electric fields, different "slices" of the beam (defined by the distance  $z$  from the beam head) may be subject to different accelerations  $F/(\gamma m)$ , where  $F$  is the force and  $\gamma$ , which may be a function of  $z$ , is the relativistic factor. In such situations, the beam may tear apart, or alternatively it may reach a non-axisymmetric equilibrium in which its cohesive self-forces counterbalance the sheared external forces. As another example, we have recently shown that there are important situations in which the beam head is in a grossly unstable equilibrium (unstable to simple zero-frequency transverse displacements) when it is propagating on-axis in a density or conductivity channel, but in which another and more stable equilibrium exists with the beam head off-axis by a prescribed amount. Thus axisymmetry is broken and the beam tends to track the channel by riding down the channel walls, rather than the channel center. Moreover, the equilibrium displacement from the channel axis increases from the head of the beam to its tail, increasing the departure from axisymmetry.

To treat problems such as these, it is necessary to develop a fully three-dimensional solution of the electromagnetic field equations; monopole/dipole

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approximations will not do. In dealing with very highly relativistic paraxial beams we shall assume, however, that it is sufficient to solve the three-dimensional form of Lee's reduced field equations<sup>3</sup>, rather than the complete Maxwell equations. This paper deals mainly with the numerical methods which we have developed to solve the field equations. To date, we have only addressed the problem of determining non-axisymmetric beam equilibria, in which the beam is specified by a prescribed density profile  $n_b(r-Y(\zeta), \zeta)$ .<sup>4</sup> Each slice of the beam is allowed transverse (non-infinitesimal) displacement  $Y(\zeta)$ , but is not allowed to distort. No particle dynamics and no time dependences have been studied, to date. Eventually we shall have to study the stability of these equilibria to internal distortions and to dynamic modes such as hose, but this can only be done after the correct equilibria have been determined, a task which is by no means trivial either computationally, mathematically or physically.

We have already discussed the code Dynasty-I, which was developed as our first cut to study the interaction between the beam and the channel. It solves the full set of Lee field equations without linearization. The radial dependence is solved by finite difference with nonuniform grids. The theta dependence is solved by fast Fourier transform. The advantage of this method is that the theta derivatives are more than second order accurate<sup>5</sup>. Unfortunately, Dynasty-I is severely time step limited because it uses an explicit scheme. This can be seen from the following: the Lee field equations are

$$\frac{\partial A}{\partial \zeta} = n \nabla_{\perp}^2 (A + \phi) + n J_b,$$

$$\nabla_{\perp}^2 \frac{\partial A}{\partial \zeta} = (\nabla \sigma) \cdot (\nabla \phi) + \sigma \nabla_{\perp}^2 \phi,$$

where  $\eta \equiv \sigma^{-1}$ . Both  $\eta$  and  $\sigma$  behave like diffusion coefficients in the equations. Near the head of the beam  $\eta$  is very large, but near the tail  $\sigma$  is very large. The difference can easily be three orders of magnitude. So any explicit scheme that takes care of the solution at one end will have severe time step limitations at the other end.

Recently we have developed a new code called Dynasty II. It is similar to Dynasty I except it solves the field equations implicitly and so the time step limitation is removed. This code involves the solution of a bi-tridiagonal system with complex matrix coefficients. The complexity in calculation is offset by the fact that for the first time, one is able to find the self consistent fields and the force acting on the beam from head to tail for arbitrary beam displacement.

We will not present the physics results investigated by Dynasty II here; they will be reported in later publications. Instead, we shall concentrate on the numerical formulation of the code. In section II, we review briefly the convolution sum, and in Sec. III we apply it to a simple diffusion equation. In Sec. IV, the method of solving the field equations is described. In Sec. V, we discuss some applications.



## II. Review of Convolution Coding

Let  $A(\theta)$  and  $B(\theta)$  be two functions that depend on  $\theta$ . The expansion of the product of  $A$  and  $B$  in a finite Fourier series is

$$\begin{aligned} A(\theta) B(\theta) &= \sum_{\ell=-N}^N A_{\ell} e^{-i\ell\theta} \sum_{m=-N}^N B_m e^{-im\theta} \\ &= \sum_{\ell=-N}^N \sum_{m=-N}^N A_{\ell} B_m e^{-i(\ell+m)\theta}. \end{aligned} \quad (1)$$

Let  $n = \ell + m$ , then (1) becomes

$$A B = \sum_{m=-N}^N \sum_{n=m-N}^{m+N} A_{n-m} B_m e^{-in\theta}. \quad (2)$$

We want to convert (2) to a form like  $\sum_{n=-N}^N ( ) e^{-in\theta}$ . Breaking the double sum to two sums and interchanging the direction of integration, we obtain

$$A B = \left[ \sum_{n=-2N}^0 \sum_{m=-N}^{n+N} \right] + \left[ \sum_{n=0}^{2N} \sum_{m=n-N}^N \right] A_{n-m} B_m e^{-in\theta}. \quad (3)$$

Equation (3) can be approximated by dropping the higher order modes, i.e. we replace  $\sum_{n=-2N}^{2N}$  by  $\sum_{n=-N}^N$ . This can be justified, provided that the magnitude of a transform goes down rapidly with higher mode number. Equation (3) becomes

$$A(\theta) B(\theta) \approx \left[ \sum_{n=-N}^0 \sum_{m=-N}^{n+N} \right] + \left[ \sum_{n=0}^N \sum_{m=n-N}^N \right] A_{n-m} B_m e^{-in\theta}. \quad (4)$$

### III. Implicit Formulation of a Diffusion Equation by the Transform Method

The RHS of a diffusion equation,

$$\frac{\partial A}{\partial \zeta}(r, \theta) = \eta(r, \theta) L[A(r, \theta)] \quad (5)$$

with  $L \equiv \nabla_{\perp}^2$ , can be expressed as

$$\text{RHS} = \left\{ \left[ \sum_{n=-N}^0 \sum_{m=-N}^{n+N} \right] + \left[ \sum_{n=0}^N \sum_{m=n-N}^N \right] \right\} \eta_{n-m}(\text{LA})_m e^{-in\theta}. \quad (6)$$

In matrix form, (6) becomes (dropping  $e^{-in\theta}$ )

$$\left[ \begin{array}{ccccccc} \eta_0(\text{LA})_{-N} & + & \dots & + & \eta_{-N}(\text{LA})_0 & + & 0 \\ \eta_1(\text{LA})_{-N} & + & \eta_0(\text{LA})_{-N+1} & + & \dots & + & \eta_{-N}(\text{LA})_1 + 0 \\ \vdots & & \vdots & & \vdots & & \vdots \\ \eta_N(\text{LA})_{-N} & + & \eta_{N-1}(\text{LA})_{-N+1} & + & \dots & + & \eta_0(\text{LA})_0 + \dots + \eta_{-N}(\text{LA})_N \\ & & & & \vdots & & \vdots \\ & & & & \vdots & & \vdots \\ & & & & \vdots & & \vdots \\ & & & & \eta_N(\text{LA})_0 & + & \dots + \eta_0(\text{LA})_N \end{array} \right] \quad (7)$$

A typical  $(\text{LA})_m$  term can be expanded to

$$(\text{LA})_m = \left( \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} - \frac{m^2}{r^2} \right) A_m(r). \quad (8)$$

Using a nonuniform grid in radial direction, i.e. using uniform spacing in  $y$ , where

$$y^2 \equiv r,$$

$$2ydy = dr,$$

we can rewrite (8) as

$$(LA)_m = \left( \frac{1}{4y^3} \frac{\partial}{\partial y} y \frac{\partial}{\partial y} - \frac{m^2}{y} \right) A_m(r). \quad (9)$$

In finite different form, (9) is

$$(LA)_m = \alpha_i A_{i+1,m} + \beta_{im} A_{i,m} + \gamma_i A_{i-1,m} \quad (10)$$

where

$$\alpha_i = y_{i+1/2} / (4 y_i^3 \Delta^2), \quad (11)$$

$$\beta_{im} = - \left( \frac{1}{2 y_i^2 \Delta^2} + \frac{m^2}{y_i^4} \right), \quad (12)$$

$$\gamma_i = \frac{y_{i-1/2}}{4 y_i^3 \Delta^2}, \quad (13)$$

$$\Delta = y_{i+1} - y_i, \quad (14)$$

Substitute (10) in (7), we obtain for the first row

$$\eta_0 (\alpha_i A_{i+1,-N} + \beta_{i,-N} A_{i,-N} + \gamma_i A_{i-1,-N})$$

$$+ \eta_{-1} (\alpha_i A_{i+1,-N+1} + \beta_{i,-N+1} A_{i,-N+1} + \gamma_i A_{i-1,-N+1}) + \dots$$

$$\begin{aligned}
& + n_{-N} (\alpha_1 A_{i+1,0} + \beta_{1,0} A_{i,0} + \gamma_1 A_{i-1,0}) \\
& = \alpha_1 (n_0, n_{-1} \dots n_{-N}) \begin{bmatrix} A_{-N} \\ A_{-N+1} \\ \vdots \\ A_0 \end{bmatrix}_{i+1} + (n_0 \beta_{1,-N}, n_{-1} \beta_{1,N+1} \dots n_{-N} \beta_{1,0}) \begin{bmatrix} A_{-N} \\ A_{-N+1} \\ \vdots \\ A_0 \end{bmatrix}_1 \\
& \quad + \gamma_1 (n_0 \dots n_{-N}) \begin{bmatrix} A_{-N} \\ A_{-N+1} \\ \vdots \\ A_0 \end{bmatrix}_{i-1}
\end{aligned} \tag{15}$$

Similarly we can rewrite all the rows in (7) in a form such as (15).

Consequently, the RHS of (5) can be expressed

$$\bar{R}_i \bar{A}_{i+1} + \bar{S}_i \bar{A}_i + \bar{T}_i \bar{A}_{i-1} \tag{16}$$

where  $\bar{A}_{i+1}$ ,  $\bar{A}_i$ ,  $\bar{A}_{i-1}$  are column matrices with the first element equal to  $A_{-N}$  and the last element equal to  $A_N$ , and  $\bar{R}_i$ ,  $\bar{S}_i$ ,  $\bar{T}_i$  are banded matrices with

$$\bar{R}_i = \bar{\alpha}_i \bar{n}_i, \tag{17}$$

$$\bar{S}_i = \bar{n}_i \bar{\beta}_i, \tag{18}$$

$$\bar{T}_i = \bar{\gamma}_i \bar{n}_i, \tag{19}$$

$$\bar{\eta}_i = \begin{bmatrix} \eta_0 & \eta_{-1} & \dots & \eta_{-N} & & & \\ & \eta_1 & \eta_0 & \eta_{-1} & \dots & & 0 \\ & & \eta_2 & \eta_1 & \eta_0 & \dots & \\ & \eta_N & \dots & & \eta_0 & \dots & \eta_{-N} \\ & & & & & & \\ 0 & & & \eta_N & \eta_{N-1} & \dots & \eta_0 & \eta_{-1} \\ & & & & & & \eta_N & \dots & \eta_1 & \eta_0 \end{bmatrix},$$

and  $\bar{\alpha}_i$  and  $\bar{\gamma}_i$  are diagonal matrices with their elements equal to  $\alpha_i$  and  $\gamma_i$  respectively.  $\bar{\beta}_i$  is also a diagonal matrix, but each element is different i.e.

$$\bar{\beta}_i = \begin{bmatrix} & & & & 0 \\ & \beta_{i,-N} & & & \\ & & \ddots & & \\ & & & \beta_{i,0} & \\ 0 & & & & \beta_{i,N} \end{bmatrix}. \quad (20)$$

Going back to (5), we finally obtain

$$\bar{A}_i^{n+1} - \bar{A}_i^n = \frac{\delta}{2} \left[ \bar{R}_i \bar{A}_{i+1}^{n+1} + \bar{S}_i \bar{A}_i^{n+1} + \bar{T}_i \bar{A}_{i-1}^{n+1} \right]$$

$$+ \frac{\delta}{2} \left[ \bar{R}_i \bar{A}_{i+1}^n + \bar{S}_i \bar{A}_i^n + \bar{T}_i \bar{A}_{i-1}^n \right], \quad (21)$$

$$\delta \equiv \zeta^{n+1} - \zeta^n \quad (22)$$

The scheme described in (21) is similar to a Crank-Nicholson method and is  $O(\delta)^2$  and  $O(\Delta)^2$ . Now we are ready to tackle the Lee field equations, as discussed in the next section.

#### IV. Implicit Formulation of the Lee Field Equations by Fourier Transform

We can cast Lee's field equations<sup>3</sup> in to the following form:

$$\frac{\partial A}{\partial \zeta} = \eta \nabla_{\perp}^2 (A + \phi) + \eta J_b \quad (23)$$

$$\nabla_{\perp}^2 \frac{\partial A}{\partial \zeta} = (\nabla \sigma) \cdot (\nabla \phi) + \sigma \nabla_{\perp}^2 \phi \quad (24)$$

where  $\eta \equiv \sigma^{-1}$ . Our aim here is to reduce (23) and (24) to a bi-tridiagonal system. The conductivity equation in normalized unit is

$$\frac{\partial \sigma}{\partial \zeta} = K_1 J_b + v_1 \sigma - K_2 p \sigma^2,$$

where

$$K_1 = 1.4653$$

$$K_2 = 1.7 \times 10^{-5}$$

$$v_1 = \frac{APS^3}{1 + BS + CS^2 + DS^3},$$

$$S = E^2/p^2,$$

$$A = 1.423 \times 10^{-4},$$

$$B = 9.179 \times 10^{-6},$$

$$C = 2.656 \times 10^{-10},$$

$$D = 2.820 \times 10^{-17}.$$

E and P are the local electric field and pressure respectively. The equation for  $\sigma$  or  $\eta$  is not solved implicitly because it includes nonlinear terms due to avalanche and recombination, but we can always time-center the  $\sigma$  equation to ensure accuracy to  $O(\delta)^2$ . For simplicity, let us define

$$\sigma' \equiv \frac{\partial \sigma}{\partial r} \quad (25)$$

$$\sigma'' \equiv \frac{\partial \sigma}{\partial \theta} \quad (26)$$

(24) can be rewritten as

$$\nabla_{\perp}^2 \frac{\partial A}{\partial \zeta} = c' \frac{\partial \phi}{\partial r} + \frac{\sigma''}{r^2} \frac{\partial \phi}{\partial \theta} + \sigma \nabla^2 \phi \quad (27)$$

We shall now transform (23) and (27) term by term. From (23), we have

$$\frac{\partial A}{\partial \zeta} = (\bar{A}_i^{n+1} - \bar{A}_i^n) \delta^{-1}, \quad (28)$$

$$\begin{aligned} \eta \nabla_{\perp}^2 A &= \frac{1}{2} [\bar{R}_i \bar{A}_{i+1}^{n+1} + \bar{S}_i \bar{A}_i^{n+1} + \bar{T}_i \bar{A}_{i-1}^{n+1}] \\ &+ \frac{1}{2} [\bar{R}_i \bar{A}_{i+1}^n + \bar{S}_i \bar{A}_i^n + \bar{T}_i \bar{A}_{i-1}^n], \end{aligned} \quad (29)$$

and

$$\eta \nabla_{\perp}^2 \phi = \frac{1}{2} [\bar{R}_i \phi_{i+1}^{n+1} + \bar{S}_i \phi_i^{n+1} + \bar{T}_i \phi_{i-1}^{n+1}]$$



$$+ \frac{1}{2} [\bar{R}_i \bar{\phi}_{i+1}^n + \bar{S}_i \bar{\phi}_i^n + \bar{T}_i \bar{\phi}_{i-1}^n]. \quad (30)$$

The term  $(\eta J_b)^{n+1/2}$  is calculated explicitly. From (27), we have

$$\begin{aligned} \nabla_{\perp}^2 \frac{\partial A}{\partial \zeta} &= \delta^{-1} \{ [\bar{\alpha}_i \bar{A}_{i+1}^{n+1} + \bar{\beta}_i \bar{A}_i^{n+1} + \bar{\gamma}_i \bar{A}_{i-1}^{n+1}] \\ &\quad - [\bar{\alpha}_i \bar{A}_{i+1}^n + \bar{\beta}_i \bar{A}_i^n + \bar{\gamma}_i \bar{A}_{i-1}^n] \}, \end{aligned} \quad (31)$$

$$\sigma \frac{\partial \phi}{\partial r} = (8y_i \Delta)^{-1} \bar{\sigma}_i [(\bar{\phi}_{i+1} - \bar{\phi}_{i-1})^{n+1} + (\bar{\phi}_{i+1} - \bar{\phi}_{i-1})^n], \quad (32)$$

and

$$\frac{\sigma''}{r^2} \frac{\partial}{\partial \theta} \phi = - \frac{\bar{i}}{2y_i^4} \bar{\sigma}_i \bar{m} [\bar{\phi}_i^{n+1} + \bar{\phi}_i^n], \quad (33)$$

where  $\bar{i} \equiv (-1)^{1/2}$  to avoid confusion with index  $i$ . Also

$$\begin{aligned} \sigma \nabla_{\perp}^2 \phi &= \frac{1}{2} [\bar{R}_i \bar{\phi}_{i+1}^{n+1} + \bar{S}_i \bar{\phi}_i^{n+1} + \bar{T}_i \bar{\phi}_{i-1}^{n+1}] \\ &\quad + \frac{1}{2} [\bar{R}_i \bar{\phi}_{i+1}^n + \bar{S}_i \bar{\phi}_i^n + \bar{T}_i \bar{\phi}_{i-1}^n]. \end{aligned} \quad (34)$$

In the above,  $\bar{m}$  is a diagonal matrix with element runs from  $-N$  to  $N$ , and  $\bar{R}_i$ ,  $\bar{S}_i$  and  $\bar{T}_i$  are defined in (17)-(19) with  $\bar{n}_i$  replaced by  $\bar{\sigma}_i$ . Putting (28)-(30) in (23) and (31)-(34) in (27), we get the desired bi-tridiagonal form:

$$a_i^{(1)} \bar{A}_{i-1} + a_i^{(2)} \bar{\phi}_{i-1} + b_i^{(1)} \bar{A}_i + b_i^{(2)} \bar{\phi}_i + c_i^{(1)} \bar{A}_{i+1} + c_i^{(2)} \bar{\phi}_{i+1} = d_i^{(1)} \quad (35)$$

$$a_i^{(3)} \bar{A}_{i-1} + a_i^{(4)} \bar{\phi}_{i-1} + b_i^{(3)} \bar{A}_i + b_i^{(4)} \bar{\phi}_i + c_i^{(3)} \bar{A}_{i+1} + c_i^{(4)} \bar{\phi}_{i+1} = d_i^{(2)} \quad (36)$$

where

$$a_i^{(1)} = -\frac{\delta \bar{T}_1}{2}, \quad (37)$$

$$a_i^{(2)} = -\frac{\delta \bar{T}_1}{2}, \quad (38)$$

$$a_i^{(3)} = \bar{\gamma}_1, \quad (39)$$

$$a_i^{(4)} = \frac{\delta \bar{\sigma}_1}{8y_1 \Delta} - \frac{\delta \bar{T}_1}{2}, \quad (40)$$

$$b_i^{(1)} = \bar{I} - \frac{\delta \bar{S}_1}{2}, \quad (41)$$

$$b_i^{(2)} = -\frac{\delta \bar{S}_1}{2}, \quad (42)$$

$$b_i^{(3)} = \bar{\beta}_1, \quad (43)$$

$$b_i^{(4)} = \frac{\bar{I}}{2} \frac{\delta}{y_1^4} \bar{\sigma}_1 - \frac{\delta \bar{S}_1}{2}, \quad (44)$$

$$c_i^{(1)} = -\frac{\delta}{2} \bar{R}_1, \quad (45)$$

$$c_i^{(2)} = -\frac{\delta}{2} \bar{R}_1, \quad (46)$$

$$c_i^{(3)} = \bar{\alpha}_1, \quad (47)$$

$$c_i^{(4)} = -\frac{\delta \bar{\sigma}_i}{8y_i \Delta} + \frac{\delta \bar{R}_i}{2}, \quad (48)$$

$$d_i^{(1)} = \frac{\delta}{2} \bar{R}_i A_{i+1}^n + \left( I + \frac{\delta \bar{S}_i}{2} \right) A_i^n + \frac{\delta \bar{T}_i}{2} A_{i-1}^n \\ + \frac{\delta}{2} (\bar{R}_i \phi_{i+1}^n + \bar{S}_i \phi_i^n + \bar{T}_i \phi_{i-1}^n) + (\eta J_b)^{n+1/2} \delta, \quad (49)$$

$$d_i^{(2)} = \bar{\alpha}_i \bar{A}_{i+1}^n + \bar{\beta}_i \bar{A}_i^n + \bar{\gamma}_i \bar{A}_{i-1}^n - \left( \frac{\delta \bar{\sigma}_i}{8y_i \Delta} + \frac{\delta \bar{R}_i}{2} \right) \bar{\phi}_{i+1}^n \\ - \left( \frac{\bar{I} \delta}{2y_i} \bar{\sigma}_i^n - \frac{\delta \bar{S}_i}{2} \right) \bar{\phi}_i^n - \left( \frac{\delta \bar{\sigma}_i}{8y_i \Delta} - \frac{\delta \bar{T}_i}{2} \right) \bar{\phi}_{i-1}^n, \quad (50)$$

where  $\bar{I}$  is an unit diagonal matrix. The inversion method of the bi-tridiagonal system is given in the appendix.

We use conducting boundary conditions at the outer boundary i.e.

$$A_m(R) = 0, \\ \phi_m(R) = 0, \quad (51)$$

for all  $m$ . Near the origin, we require both the electric and magnetic fields be finite and continuous. Noting that for small  $r$ ,

$$A_m(r), \phi_m(r) \propto r^m, \quad (52)$$

we have at once

$$A_m(0) = 0 = \phi_m(0), \quad \text{for } m > 1, \quad (53)$$

$$\frac{\partial A_m(0)}{\partial r} = 0 = \frac{\partial \phi_m(0)}{\partial r}, \quad \text{for } m = 0. \quad (54)$$

## V. Applications

To date, Dynasty II has been used only to calculate beam equilibria, using a simple envelope model of the beam. The beam current density is specified to be of the form

$$J_b(r, \zeta) = f \left[ \frac{|r - Y(\zeta)|}{a(\zeta)} \right],$$

where  $f$  is a specified radial profile shape (usually Bennett),  $a(\zeta)$  is the beam radius, and  $Y(\zeta)$  is the transverse displacement of the slice. In most cases, we have specified  $a(\zeta)$  and used the code, iterating back in  $\zeta$  from the beam head, to calculate the equilibrium value of  $Y(\zeta)$ . We have already mentioned briefly some applications of the code in Sec. I. We shall elaborate on these areas in some detail here.

Previous work<sup>6</sup> has shown that in the electrostatic regime at the beam head if the beam radius  $r_b$  is smaller than the conductivity channel  $r_c$ , the beam is attracted towards the channel axis. On the other hand, if  $r_b > r_c$  as is usually true in the beam head, we have shown that when the beam resides on the channel axis, each slice will see an average force repelling it from the channel. But in the electrostatic regime, if the beam is displaced sufficiently from the channel it should see an electric dipole attracting it back toward the channel. Using Dynasty II to study this problem, we found that when the beam displacement is of the order of the beam radius and  $r_b > r_c$ , a stable equilibrium exists with the beam off axis, i.e.  $Y \neq 0$ . The equilibrium displacement from the channel axis increases from the head of the beam to its tail. A detailed description of these results will be presented in a later report.

When an external electric or magnetic force  $F$  acts on the beam, each beam slice will be subject to a different acceleration  $F/m\gamma$  if  $\gamma$  is a function of  $\zeta$ .

If the shear in  $F(\zeta)/m\gamma(\zeta)$  is strong compared to the restoring force due to the beam's self-pinch, the beam will tear. The break-up will continue along  $\zeta$  until for some value  $\zeta_c$  the restoring force becomes stronger than the sheared external force; from this point on, i.e.  $\zeta > \zeta_c$ , the beam will hold together. Once the "guiding point"  $\zeta_c$  is determined, we can find (a) the deflection due to the force  $F$ , (b) what portion of the beam is torn out, and (c) the value of  $\gamma(\zeta_c)$ , i.e., the energy of that part of the beam actually guides the rest of the beam. If  $\gamma(\zeta_c)$  differs substantially from  $\gamma(\zeta = 0)$ , then a problem may occur in aiming the beam subject to the influence of this external force. This work is still in progress and the results will be reported in a later report.

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## Appendix

The algorithm for inverting a bi-triangular system with scalar coefficients is well known.<sup>7</sup> We shall generalize it to include matrix coefficients. The equations are

$$a_i^{(1)}u_{i-1} + a_i^{(2)}v_{i-1} + b_i^{(1)}u_i + b_i^{(2)}v_i + c_i^{(1)}u_{i+1} + c_i^{(2)}v_{i+1} = d_i^{(1)}$$

and

$$a_i^{(3)}u_{i-1} + a_i^{(4)}v_{i-1} + b_i^{(3)}u_i + b_i^{(4)}v_i + c_i^{(3)}u_{i+1} + c_i^{(4)}v_{i+1} = d_i^{(1)},$$

for  $1 < i < R$ , with  $a_i^{(m)} = c_R^{(m)} = 0$  for  $1 < m < 4$ .

The algorithm is as follows:

First compute

$$\beta_i^{(1)} = b_i^{(1)} - a_i^{(1)}\lambda_{i-1}^{(1)} - a_i^{(2)}\lambda_{i-1}^{(3)},$$

$$\beta_i^{(2)} = b_i^{(2)} - a_i^{(1)}\lambda_{i-1}^{(2)} - a_i^{(2)}\lambda_{i-1}^{(4)},$$

$$\beta_i^{(3)} = b_i^{(3)} - a_i^{(3)}\lambda_{i-1}^{(1)} - a_i^{(4)}\lambda_{i-1}^{(3)},$$

$$\beta_i^{(4)} = b_i^{(4)} - a_i^{(3)}\lambda_{i-1}^{(2)} - a_i^{(4)}\lambda_{i-1}^{(4)},$$

with  $\beta_1^{(m)} = b_1^{(m)}$  for  $1 < m < 4$ ,

and

$$\delta_i^{(1)} = d_i^{(1)} - a_i^{(1)} \gamma_{i-1}^{(1)} - a_i^{(2)} \gamma_{i-1}^{(2)},$$

$$\delta_i^{(2)} = d_i^{(2)} - a_i^{(3)} \gamma_{i-1}^{(1)} - a_i^{(4)} \gamma_{i-1}^{(2)},$$

with  $\delta_i^{(1)} = d_i^{(1)}$  and  $\delta_i^{(2)} = d_i^{(2)}$ , and

$$\mu_i^{(1)} = \beta_i^{(2)-1} \beta_i^{(1)} - \beta_i^{(4)-1} \beta_i^{(3)},$$

$$\mu_i^{(2)} = \beta_i^{(1)-1} \beta_i^{(2)} - \beta_i^{(3)-1} \beta_i^{(4)},$$

$$\lambda_i^{(1)} = \mu_i^{(1)-1} (\beta_i^{(2)-1} c_i^{(1)} - \beta_i^{(4)-1} c_i^{(3)}),$$

$$\lambda_i^{(2)} = \mu_i^{(1)-1} (\beta_i^{(2)-1} c_i^{(2)} - \beta_i^{(4)-1} c_i^{(4)}),$$

$$\lambda_i^{(3)} = \mu_i^{(2)-1} (\beta_i^{(1)-1} c_i^{(1)} - \beta_i^{(3)-1} c_i^{(3)}),$$

$$\lambda_i^{(4)} = \mu_i^{(2)-1} (\beta_i^{(1)-1} c_i^{(2)} - \beta_i^{(3)-1} c_i^{(4)}),$$

$$\gamma_i^{(1)} = \mu_i^{(1)-1} (\beta_i^{(2)-1} \delta_i^{(1)} - \beta_i^{(4)-1} \delta_i^{(2)}),$$

$$\gamma_i^{(2)} = \mu_i^{(2)-1} (\beta_i^{(1)-1} \delta_i^{(1)} - \beta_i^{(3)-1} \delta_i^{(2)}).$$

Using

$$u_R = \gamma_R^{(1)},$$

$$v_R = \gamma_R^{(2)},$$

we get

$$u_i = \gamma_i^{(1)} - \lambda_i^{(1)} u_{i+1} - \lambda_i^{(2)} v_{i+1},$$

$$v_i = \gamma_i^{(2)} - \lambda_i^{(3)} u_{i+1} - \lambda_i^{(4)} v_{i+1},$$

for  $(R-1) > i > 1$ .



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